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RADIATIVE TRANSFER TO OSCILLATORY HYDROMAGNETIC ROTATING FLOW OF A RAREFIED GAS PAST A HORIZONTAL FLAT PLATE

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SUMMARY

Hydromagnetic flow past an infinite horizontal plate is considered when the flow is rarefied and the temperature of the wall is high enough for radiative heat transfer to be significant. In the undisturbed flow far away from the plate, an oscillatory velocity is superimposed on a steady mean and the whole configuration is in constant rotation. When the flow is slightly rarefied, the compressible Navier–Stokes equations and the slip boundary conditions together with the general differential approximation for radiation suffice for the analytical description of the problem. If the amplitude of oscillation is small, the problem is tackled by a perturbation scheme and numerical integration. Consequences of the effect of rotation and oscillation on the flow variables are discussed.

KEY WORDS Hydromagnetic Radiating Rarefied gas flow

1. INTRODUCTION

The problem of rarefied and electrically conducting gas flow is applicable in ultrahigh-altitude aerodynamics and in fission research. On the other hand, oscillatory flow is of natural and frequent occurrence in aeronautical work with attendant problems, notably in flutter. Hence the motivation for this study.

An important parameter in the study of rarefied gas dynamics is the ratio of the mean free path of the gas, l, to a characteristic length of flow, this parameter being referred to as the Knudsen number, Kn. When Kn is zero, continuum theory is valid. However, for small Kn it is still appropriate to adopt continuum theory and, as discussed by Shidlovskiy,[1] the continuum hypothesis gives very good results even when Kn is of order $O(1)$.

The general feature of slightly rarefied gas flow past a solid body is clarified in the asymptotic theory developed in References 2–4. In brief, the theory gives a systematic derivation of the classical slip flow theory from the linearized Boltzmann equation including higher-order contributions in the Knudsen number. Thus the boundary condition for the $O(1)$ case is the no-slip condition of classical fluid dynamics. In the order $O(Kn)$ approximation the slip boundary condition gives the first-order correction to classical theory.

Hence in this study we employ the compressible Navier–Stokes equations and the slip boundary conditions, while the radiative flux term is also accommodated by adopting the general differential approximation for radiation. The whole problem is therefore reducible to differential equation form, and the formulation is given in Section 2. Section 3 is devoted to a solution of the

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basic approximation, while in Section 4 the higher approximate solutions are developed. The results of the previous three sections are discussed in Section 5.

2. MATHEMATICAL FORMULATION

We consider oscillatory flow $U_\infty(1+\varepsilon\cos\omega t')$ past a horizontal flat plate, where U_∞ is a typical velocity, ω is frequency, t' is time and ε is a parameter. The plate is maintained at a temperature T_0 which is large enough for radiation to be significant. A magnetic field H_0 is applied perpendicular to the plate in the y'-direction. The whole configuration rotates with angular velocity Ω about this perpendicular, which points in the reverse direction to gravitation g. The flow is therefore two-dimensional in the Cartesian (x', y') co-ordinate with the x'-axis lying on the plate, such that the velocity, magnetic field and radiative flux components are (u', v'), (H', H_0) and $(0, q')$.

We now make the following assumptions:

(i) The flow is slightly rarefied so that the compressible Navier–Stokes equations and the slip boundary conditions may be adopted, such that in the slip conditions χ is the reflection coefficient and λ the accommodation coefficient.

(ii) The general differential approximation for radiation may be invoked, in which case α_r is the absorption coefficient, σ_r is the Stefan–Boltzmann constant and ε_w is the emittence of the wall.

(iii) The gas is perfect such that $p'=\rho' RT$ and c_p is constant, while the square of the speed of sound is $a^2=\gamma p'/\rho'$, p' is the pressure, ρ' the density, T the temperature, R the gas constant, c_p the specific heat at constant pressure and γ the ratio of the specific heats.

(iv) The viscosity of the fluid, μ, is proportional to the temperature, while the Prandtl number $Pr(=\mu c_p/k)$ is constant, where k is the thermal conductivity.

(v) The fluid is electrically conducting with conductivity σ_c, so that in electromagnetic units the permeability of free space is unity.

Since the plate is infinite, all variables will depend on y and t alone. With this and the assumptions enumerated above, the governing non-dimensional equations of motion may be written as

$$
\begin{aligned}
&\sigma\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial y}(\rho v)=0,\\
&\rho\left(\sigma\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}-Ev\right)=\frac{\partial}{\partial y}\left(\theta\frac{\partial u}{\partial y}\right)+M^2\frac{\partial h}{\partial y},\\
&\rho\left(\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial y}+Eu\right)=-\frac{\partial p}{\partial y}+\frac{\partial}{\partial y}\left(\theta\frac{\partial v}{\partial y}\right)-M^2h\frac{\partial h}{\partial y}-F_\infty\rho,\\
&\rho\left(\sigma\frac{\partial\theta}{\partial t}+v\frac{\partial\theta}{\partial y}\right)=\frac{1}{Pr}\frac{\partial}{\partial y}\left(\theta\frac{\partial\theta}{\partial y}\right)-\frac{\partial q}{\partial y},\\
&\frac{\partial^2 h}{\partial y^2}+D_m\left(\frac{\partial u}{\partial y}-\sigma\frac{\partial h}{\partial y}\right)=0,\\
&\frac{\partial^2 q}{\partial y^2}-\frac{3}{N^2}q-\frac{3}{B_0}\theta^3\frac{\partial\theta}{\partial y}=0,\\
&p=\frac{1}{\gamma M_\infty^2}\rho\theta,
\end{aligned}
\tag{1}
$$

such that

$$y=\frac{2-\chi}{\chi}\frac{5\pi}{16}Kn\frac{\partial u}{\partial y}, \qquad v=0,$$

$$\frac{\theta}{\theta_0}=1+\frac{2-\lambda}{\lambda}\frac{75\pi}{128}Kn\frac{1}{\theta}\frac{\partial\theta}{\partial y}+\frac{5}{24}\sqrt{\left(\frac{\gamma\pi}{2}\right)}KnM_\infty\frac{1}{\theta^{1/2}}\frac{\partial v}{\partial y}, \tag{2}$$

$$\frac{p}{p_0}=1+\frac{2-\chi}{\chi}Kn\frac{1}{\theta}\frac{\partial\theta}{\partial y}+\frac{5}{6}\sqrt{\left(\frac{\gamma\pi}{2}\right)}KnM_\infty\frac{1}{\theta^{1/2}}\frac{\partial f}{\partial y},$$

$$\left(\frac{1}{\varepsilon_w}-\frac{1}{2}\right)q-\frac{N}{4}\frac{\partial q}{\partial y}=\frac{3}{16}\frac{N}{B_0}\left(\theta_0^4-\theta^4\right), \qquad h=0 \quad \text{on } y=0;$$

$$u=1+\varepsilon\cos t, \qquad (v, q, h)\to 0, \qquad (\theta, \rho)\to 1 \quad \text{as } y\to\infty, \tag{3}$$

where subscript ∞ denotes uniform flow conditions. We have adopted the general differential approximation for radiation for a grey gas as given by Cheng.[5]

We have introduced the following non-dimensional quantities:

$$t=\omega t', \qquad y=\frac{U_\infty y'}{v_\infty}, \qquad (u, v)=(u', v')/U_\infty,$$

$$(\theta, \theta_0)=(T, T_0)/T_\infty, \qquad \rho=\rho'/\rho_\infty, \qquad h=H'/H_0,$$

$$p=p'/\rho_\infty U_\infty^2, \qquad q=q'/\rho_\infty c_p U_\infty T_\infty, \qquad E=v_\infty\Omega/U_\infty^2, \tag{4}$$

$$\sigma=v_\infty\omega/U_\infty^2, \qquad D_m=v_\infty\sigma_e, \qquad M=H_0/\rho_\infty^{1/2}U_\infty,$$

$$F_\infty=v_\infty g/U_\infty^3, \qquad M_\infty=U_\infty/a_\infty, \qquad N=U_\infty/v_\infty\alpha_r,$$

$$B_0=3\rho_\infty c_p U_0^2/16\sigma_r\alpha_r v_\infty T_\infty^3, \qquad Kn=U_\infty l/v_\infty.$$

The kinematic viscosity of the fluid v is defined as $v=\mu/\rho$.

The slip boundary conditions on the velocity, temperature and pressure are as given in Shidlovskiy,[1] while that on the radiative flux is due to Cess.[6] The condition on the magnetic field assumes that the wall is a perfect insulator. Apart from Pr and Kn, the problem also depends on the oscillatory parameter σ, the rotation parameter E, the ratio of kinematic viscosity to the magnetic diffusivity D_m, the magnetic parameter M, the buoyancy parameter F_∞, the Mach number M_∞ and the radiation parameters N and B_0. The mathematical statement of the problem is to solve (1) subject to (2) and (3).

3. PERTURBATION AND LEADING SOLUTION

To tackle the problem posed in the previous section, we introduce the expansion (assuming ε is small)

$$u=u^{(0)}(y)+\tfrac{1}{2}\varepsilon[u^{(1)}(y)e^{it}+\tilde{u}^{(1)}e^{-it}]+\ \ldots \tag{5a}$$

for all the dependent variables except v, while for v we write

$$v=\tfrac{1}{2}\varepsilon[v^{(1)}(y)e^{it}+\tilde{v}^{(1)}e^{-it}]+\ \ldots. \tag{5b}$$

Substituting (5) in (1)–(3), the order $O(1)$ problem is

$$0=\frac{1}{Pr}\frac{\mathrm{d}}{\mathrm{d}y}\left(\theta^{(0)}\frac{\mathrm{d}\theta^{(0)}}{\mathrm{d}y}\right)-\frac{\mathrm{d}q^{(0)}}{\mathrm{d}y},\qquad \frac{\mathrm{d}^2q^{(0)}}{\mathrm{d}y^2}-\frac{3}{N^2}q^{(0)}-\frac{3}{B_0}\theta^{(0)3}\frac{\mathrm{d}\theta^{(0)}}{\mathrm{d}y}=0, \tag{6}$$

$$\frac{\mathrm{d}}{\mathrm{d}y}\left(\theta^{(0)}\frac{\mathrm{d}u^{(0)}}{\mathrm{d}y}\right)+M^2\frac{\mathrm{d}h^{(0)}}{\mathrm{d}y}=0,\qquad \frac{\mathrm{d}^2h^{(0)}}{\mathrm{d}y^2}+D_{\mathrm{m}}\frac{\mathrm{d}u^{(0)}}{\mathrm{d}y}=0, \tag{7}$$

$$Eu^{(0)}\rho^{(0)}=-\frac{\mathrm{d}p^{(0)}}{\mathrm{d}y}-M^2h^{(0)}\frac{\mathrm{d}h^{(0)}}{\mathrm{d}y}-F_\infty\rho^{(0)},\qquad p^{(0)}=\frac{1}{\gamma M_\infty^2}\rho^{(0)}\theta^{(0)}, \tag{8}$$

where

$$\begin{gathered}
u^{(0)}=\frac{2-\chi}{\chi}\frac{5\pi}{16}Kn\frac{\mathrm{d}u^{(0)}}{\mathrm{d}y},\qquad v^{(0)}=0,\\
\frac{\theta^{(0)}}{\theta_0}=1+\frac{2-\lambda}{\lambda}\frac{75\pi}{128}Kn\frac{1}{\theta^{(0)}}\frac{\mathrm{d}\theta^{(0)}}{\mathrm{d}y},\\
\frac{p}{p_0}=1+\frac{2-\chi}{\chi}Kn\frac{1}{\theta^{(0)}}\frac{\mathrm{d}\theta^{(0)}}{\mathrm{d}y},\qquad h^{(0)}=0,\\
\left(\frac{1}{\varepsilon_{\mathrm{w}}}-\frac{1}{2}\right)q^{(0)}-\frac{N}{4}\frac{\mathrm{d}q^{(0)}}{\mathrm{d}y}=\frac{3}{16}\frac{N}{B_0}(\theta_0^4-\theta^4)\quad\text{on } y=0;
\end{gathered} \tag{9}$$

$$u^{(0)}=1,\qquad (q^{(0)},h^{(0)})\to 0,\qquad (\theta^{(0)},\rho^{(0)})\to 1\quad\text{as } y\to\infty. \tag{10}$$

The solutions of equation (6) subject to the appropriate conditions in (9) and (10) have been discussed exhaustively by Bestman.[7] All three possible cases were considered.

(i) *Optically thick case*

$$\frac{\theta^{(0)2}-1}{\theta^{(0)2}+1}=\left(\frac{\theta^{*2}-1}{\theta^{*2}+1}\right)\exp\left(-\frac{N^2Pr}{B_0}y\right),\qquad \frac{\mathrm{d}\theta^{(0)}}{\mathrm{d}y}=-\frac{N^2Pr}{4B_0}\frac{(\theta^{(0)4}-1)}{\theta^{(0)}},$$

where $\theta^*=\theta^{(0)}(0)$, which is obtained from the equation

$$\theta^{*4}+\alpha^{(0)}\theta^{*3}-\beta^{(0)}\theta^{*2}-1=0,$$

such that

$$Kn^{(0)}=\frac{2-\lambda}{\lambda}\frac{75\pi}{128}Kn,\qquad q^{(0)}=\frac{N^2Pr}{4B_0}Kn^{(0)},$$

$$\alpha^{(0)}=1/q^{(0)}\theta^{(0)},\qquad \beta^{(0)}=1/q^{(0)}.$$

(ii) *Optically thin case*

$$\theta^{(0)}=\wp^{1/2}\left(\frac{Pr^{1/2}}{2B_0^{1/2}}y+c,12,8\right),\qquad c=\int_\infty^{\theta^{*2}}\frac{\mathrm{d}s}{\sqrt{(4s^2-12s-8)}},$$

$$\frac{\mathrm{d}\theta^{(0)}}{\mathrm{d}y}=\frac{Pr^{1/2}}{4B_0^{1/2}}(4\theta^{(0)6}-12\theta^{(0)2}-8),$$

where θ^* follows from the equation

$$(4c^{(0)}-1)\theta^{*6}+2\theta_0\theta^{*5}+\theta_0^2\theta^{*4}-12c^{(0)}\theta^{*2}-8c^{(0)}=0,$$

$c^{(0)1/2}=Kn^{(0)}Pr^{1/2}\theta_0/4B_0^{1/2}$ and $\wp$ is the Weierstrass elliptic function.

(iii) *Arbitrary optical thickness*

$$q^{(0)}=\frac{1}{Pr}\theta^{(0)}\frac{d\theta^{(0)}}{dy},\qquad q^{(0)2}=\frac{1}{4PrB_0}\theta^{(0)6}+\frac{3}{4Pr^2N^2}\theta^{(0)4}+C\theta^{(0)2}+D,$$

where

$$D=-C-B_{\rm n}^{(0)},\qquad B_{\rm n}^{(0)}=-\frac{1}{4PrB_0}-\frac{3}{4Pr^2B_0^2},$$

$$C(\theta^{*2}-1)=\frac{1}{Pr^2Kn^{(0)2}}\theta^{*4}\left(\frac{\theta^*}{\theta_0}-1\right)-\frac{1}{4PrB_0}\theta^{*6}-\frac{3}{4Pr^2N^2}\theta^{*4}+B_{\rm n}^{(0)},$$

$$a_0\theta^{*12}+a_1\theta^{*11}+\ \dots\ +a_j\theta^{*12-j}+\ \dots\ +a_{11}\theta^*+a_{12}=0.$$

The coefficients of the polynomial, which are quite complicated, have been given in Reference 7 and will not be repeated here. A brief account is given in the Appendix.

Now that $\theta^{(0)}$ and $d\theta^{(0)}/dy$ are known, equations (7) become linear and with the concomitant boundary conditions could be integrated by a finite difference scheme employing central differences. With u and h known, equation (8) could be expressed as

$$\frac{d\rho^{(0)}}{dy}+\frac{1}{\theta^{(0)}}\left(\frac{d\theta^{(0)}}{dy}+\gamma M_\infty^2F_\infty+\gamma M_\infty^2E\frac{u^{(0)}}{\theta^{(0)}}\right)\rho^{(0)}=-\frac{1}{2}\gamma M_\infty^2M^2\frac{1}{\theta^{(0)}}\frac{dh^{(0)2}}{dy^2},\tag{11}$$

which is a first-order linear equation for the single unknown $\rho^{(0)}$. Integrating equation (11) and imposing the condition on the density at $y=\infty$, we can show that

$$\begin{aligned}\rho^{(0)}=&-\frac{1}{2}\gamma M_\infty^2M^2h^{(0)2}\frac{1}{\theta^{(0)}}+\frac{1}{\theta^{(0)}}\exp\left[-\gamma M_\infty^2\int_\infty^y\left(\frac{F_\infty}{\theta^{(0)}}+E\frac{u^{(0)}}{\theta^{(0)}}\right)dz\right]\\&+\frac{1}{2}\gamma^2M_\infty^4M^2\frac{1}{\theta^{(0)}}\exp\left[-\gamma M_\infty^2\int_\infty^y\left(\frac{F_\infty}{\theta^{(0)}}+E\frac{u^{(0)}}{\theta^{(0)}}\right)dz\right]\int_\infty^y h^2\left(\frac{F_\infty}{\theta^{(0)}}+E\frac{u^{(0)}}{\theta^{(0)}}\right)\\&\times\exp\left[\gamma M_\infty^2\int_0^z\left(\frac{F_\infty}{\theta^{(0)}}+E\frac{u^{(0)}}{\theta^{(0)}}\right)dz'\right]dz.\end{aligned}\tag{12}$$

The point $y=\infty$ is taken to correspond to $\theta^{(0)}(y)=1$. For the optically thick gas the integral $\int dy/\theta^{(0)}$ may be evaluated in a close form by changing the variable of integration to $\theta^{(0)}$. Thus

$$\int\frac{dy}{\theta^{(0)}}=-\frac{4B_0}{N^2Pr}\int\frac{d\theta^{(0)}}{\theta^{(0)}(\theta^{(0)4}-1)}=-\frac{4B_0}{N^2Pr}\ln\left(\frac{(\theta^{(0)}-1)^{1/4}(\theta^{(0)2}+1)^{1/2}}{\theta^{(0)}}\right)\tag{13}$$

and the labour of numerical integration of (12) is reduced, particularly if the effect of change of E is negligible in which case only single integrals are involved. It will be shown that this is so.

Finally with, $\rho^{(0)}$ now determined, the unknown pressure at the wall, p_0, can be computed from the remaining pressure jump condition. Thus

$$\frac{1}{\gamma M_\infty^2p_0}=1+\frac{2-\chi}{\chi}\frac{1}{\theta^*}\frac{d\theta^{(0)}}{dy}\bigg|_{\theta^{(0)}=\theta^*}.\tag{14}$$

The solution is now complete and the value p_0 is used in the subsequent calculation.

In this section we have employed the finite difference algorithm in obtaining solutions for the velocity and magnetic field components. Indeed, the finite difference scheme is all that will be adopted in discussing the order $O(\varepsilon)$ solutions of the subsequent section. Hence it is necessary to have a knowledge of the error involved in the use of this scheme.

Thus equations (7) could be expressed in the form

$$\theta^{(0)}\frac{d^2u^{(0)}}{dy^2}+\frac{d\theta^{(0)}}{dy}\frac{du^{(0)}}{dy}-D_mM^2u^{(0)}=-D_mM^2, \qquad h^{(0)}=D_m\int_0^y(1-u^{(0)})\,dz.$$

Setting $u^{(0)}=1+\theta^{(0)1/2}\,U^{(0)}$, these equations become

$$\frac{d^2U^{(0)}}{dy^2}+\frac{1}{4}\theta^{(0)}\left[2\theta^{(0)2}\frac{d^2\theta^{(0)}}{dy^2}+\left(\frac{d\theta^{(0)}}{dy}\right)^2\right]U^{(0)}-D_mM^2\frac{1}{\theta^{(0)}}U^{(0)}=0,$$

$$h^{(0)}=-D_m\int_0^y\theta^{(0)1/2}U^{(0)}dz.$$

We now consider the case when $D_mM^2\gg 1$, whence the equation for $U^{(0)}$ approximates to

$$\delta^2\frac{d^2U^{(0)}}{dy^2}=\frac{1}{\theta^{(0)}}U^{(0)}, \qquad \delta=\frac{1}{D_m^{1/2}M}.$$

The asymptotic solution of this last equation may be obtained by the WKB approximation. Retaining only the eikonal and transport terms, the solution satisfying the appropriate boundary condition

$$\theta^{(0)1/2}U^{(0)}=-1+\frac{2-\chi}{\chi}\frac{5\pi}{16}Kn\left(\theta^{(0)1/2}\frac{dU^{(0)}}{dy}+\frac{1}{2}\theta^{(0)-1/2}\frac{d\theta^{(0)}}{dy}U^{(0)}\right) \quad \text{on } y=0$$

is

$$U^{(0)}\sim\frac{\theta^{(0)1/2}\exp\left(-\frac{1}{\delta}\int_0^y\theta^{(0)-1/2}dz\right)}{\frac{2-\chi}{\chi}\frac{5\pi}{16}Kn\left(\frac{1}{4\theta^{*1/4}}-\frac{1}{\delta}\theta^{*1/4}+\frac{1}{2}\theta^{*1/4}\frac{d\theta^{(0)}}{dy}(0)\right)-\theta^{*3/4}}. \tag{10a}$$

Hence $h^{(0)}$ follows. All the integrals involved are evaluated by an efficient Romberg numerical scheme.

4. HIGHER APPROXIMATE SOLUTION

If we continue the expansion started in (5), we find that the governing equations for the order $O(\varepsilon)$ problem may be put in the convenient form

$$i\sigma\rho^{(1)}+\frac{d\rho^{(1)}}{dy}v^{(1)}+\rho^{(0)}\frac{dv^{(1)}}{dy}=0,$$

$$\rho^{(0)}\left(i\sigma u^{(1)}+\frac{du^{(0)}}{dy}v^{(1)}-Ev^{(1)}\right)=\frac{d}{dy}\left(\theta^{(0)}\frac{du^{(1)}}{dy}+\frac{du^{(0)}}{dy}\theta^{(1)}\right)+M^2\frac{dh^{(1)}}{dy},$$

$$i\sigma\rho^{(0)}v^{(1)}+E(\rho^{(0)}u^{(1)}+u^{(0)}\rho^{(1)})=-\frac{1}{\gamma M_\infty^2}\frac{d}{dy}(\rho^{(0)}\theta^{(1)}+\theta^{(0)}\rho^{(1)})+\frac{d}{dy}\left(\theta^{(0)}\frac{dv^{(1)}}{dy}\right)$$
$$-M^2\left(h^{(0)}\frac{dh^{(1)}}{dy}+\frac{dh^{(0)}}{dy}h^{(1)}\right)-F_\infty\rho^{(1)},$$

$$\rho^{(0)}\left(\mathrm{i}\sigma\theta^{(1)}+\frac{\mathrm{d}\theta^{(0)}}{\mathrm{d}y}v^{(1)}\right)=\frac{1}{Pr}\frac{\mathrm{d}}{\mathrm{d}y}\left(\theta^{(0)}\frac{\mathrm{d}\theta^{(1)}}{\mathrm{d}y}+\frac{\mathrm{d}\theta^{(0)}}{\mathrm{d}y}\theta^{(1)}\right)-\frac{\mathrm{d}q^{(1)}}{\mathrm{d}y}, \tag{15}$$

$$\frac{\mathrm{d}^2q^{(1)}}{\mathrm{d}y^2}-\frac{3}{N^2}q^{(1)}-\frac{3}{B_0}\left(\theta^{(0)3}\frac{\mathrm{d}\theta^{(1)}}{\mathrm{d}y}+3\theta^{(0)2}\frac{\mathrm{d}\theta^{(0)}}{\mathrm{d}y}\theta^{(1)}\right)=0,$$

$$\frac{\mathrm{d}^2h^{(1)}}{\mathrm{d}y^2}+D_{\mathrm{m}}\left(\frac{\mathrm{d}u^{(1)}}{\mathrm{d}y}-\mathrm{i}\sigma h^{(1)}\right)=0,$$

where

$$u^{(1)}=\frac{2-\chi}{\chi}\frac{5\pi}{16}Kn\frac{\mathrm{d}u^{(1)}}{\mathrm{d}y},\qquad v^{(1)}=0=h^{(1)},$$

$$\frac{\theta^{(1)}}{\theta_0}=\frac{2-\lambda}{\lambda}\frac{75\pi}{128}Kn\frac{1}{\theta^{(0)}}\left(\frac{\mathrm{d}\theta^{(1)}}{\mathrm{d}y}-\frac{1}{\theta^{(0)}}\frac{\mathrm{d}\theta^{(0)}}{\mathrm{d}y}\theta^{(1)}\right)-\frac{5}{24}\sqrt{\left(\frac{\gamma\pi}{2}\right)}KnM_\infty\frac{1}{\theta^{(0)1/2}}\frac{\mathrm{i}\sigma\rho^{(1)}}{\rho^{(0)}}, \tag{16}$$

$$\frac{1}{\gamma M_\infty^2p_0}(\rho^{(0)}\theta^{(1)}+\theta^{(0)}\rho^{(1)})=\frac{2-\chi}{\chi}Kn\frac{1}{\theta^{(0)}}\left(\frac{\mathrm{d}\theta^{(1)}}{\mathrm{d}y}-\frac{1}{\theta^{(0)}}\frac{\mathrm{d}\theta^{(0)}}{\mathrm{d}y}\theta^{(1)}\right)-\frac{5}{6}\sqrt{\left(\frac{\gamma\pi}{2}\right)}KnM_\infty\frac{1}{\theta^{(0)1/2}}\frac{\mathrm{i}\sigma\rho^{(1)}}{\rho^{(0)}},$$

$$\left(\frac{1}{\varepsilon_{\mathrm{w}}}-\frac{1}{2}\right)q^{(1)}-\frac{N}{4}\frac{\mathrm{d}q^{(1)}}{\mathrm{d}y}=-\frac{3N}{4B_0}\theta^{(0)3}\theta^{(1)}\quad\text{on } y=0,$$

$$u^{(1)}\to 1,\qquad (v^{(1)},\theta^{(1)},\rho^{(1)},h^{(1)},q^{(1)})\to 0\quad\text{as } y\to\infty.$$

In equations (16) we have replaced the $\mathrm{d}v^{(1)}/\mathrm{d}y$ term in the temperature and density boundary conditions by the continuity in (15). Equations (15) and (16) constitute a linear coupled two-point boundary value problem for the six unknowns $u^{(1)}$, $v^{(1)}$, $h^{(1)}$, $q^{(1)}$, $\rho^{(1)}$ and $\theta^{(1)}$. By expressing the unknowns in terms of their real and imaginary parts, the problem is tackled by a finite difference scheme, again using central differences.

5. DISCUSSION

In the numerical discussion of the problem we shall only consider the effect of rotation E and magnetic field M on the flow variable. The effects of the other parameters have been considered previously.[7] Hence we take $\theta_0=10$, $N=0{\cdot}33$, $B_0=1{\cdot}0$, $M_\infty=0{\cdot}2$, $F_\infty=0{\cdot}1=Kn$, $\chi=0{\cdot}9=\lambda$, $Pr=0{\cdot}71$, $\gamma=1{\cdot}4$, $\varepsilon_{\mathrm{w}}=1/2$ and $\varepsilon=0{\cdot}1$.

In Figures 1–3 the velocity component parallel to the plate, the temperature and the magnetic field are depicted for the general differential approximation for a grey gas. Thus

$$u=u^{(0)}+\varepsilon(u_{\mathrm{R}}^{(1)}\cos t-u_{\mathrm{I}}^{(1)}\sin t),\quad\text{etc.,} \tag{7}$$

where subscripts R and I represent real and imaginary parts of a complex quantity.

It is noted that an increase in the magnetic parameter M^2 causes a rise in the flow variables. However, whether the rotation rate is slow or fast, the flow variables remain essentially unaltered.

This last effect is very useful in the computation of the density $\rho^{(0)}$ in equation (12). Indeed, the E terms in the integrals involving the exponential terms may be replaced by a mean and, as shown by Bestman,[7] the error incurred in using the optically thick approximation is not much when

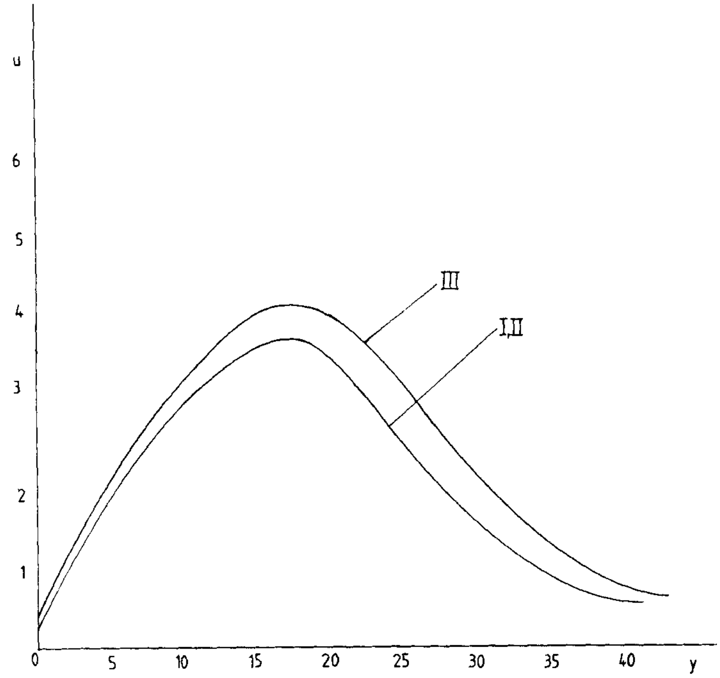

Figure 1. Velocity distribution: I, $E=0{\cdot}5$, $M^2=5$; II, $E=0{\cdot}5$, $M^2=10$; III, $E=10$, $M^2=10$

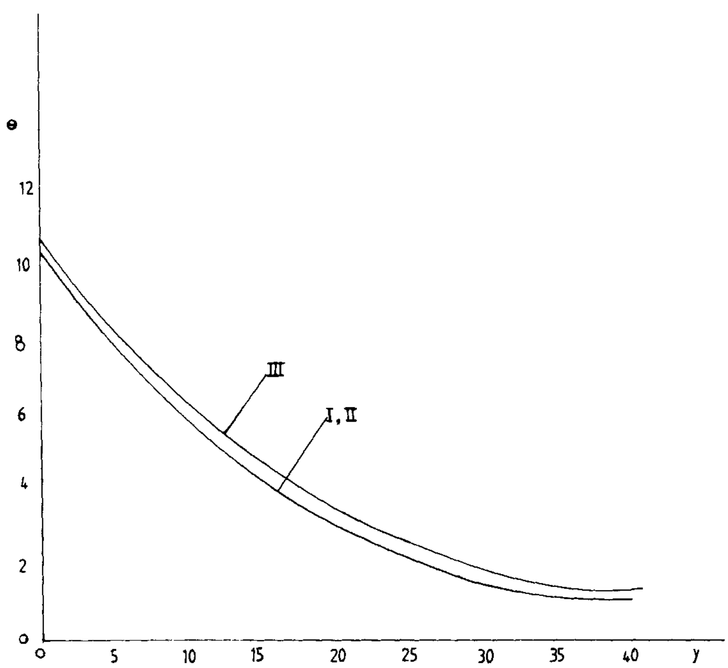

Figure 2. Temperature distribution (for key see Figure 1)

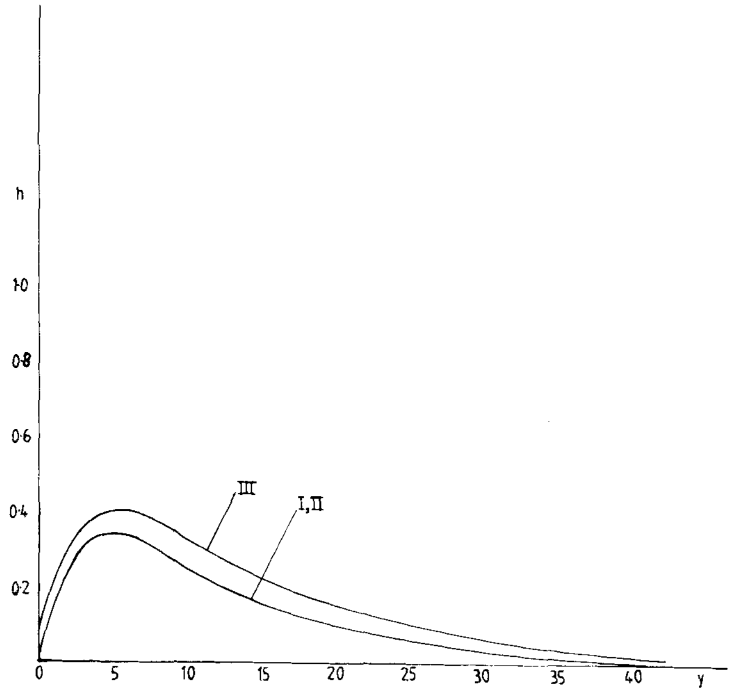

Figure 3. Magnetic field distribution (for key see Figure 1)

compared with the exact differential approximation. To be precise, this error is of the order of about 5%. The computational time saved in using these approximations is as much as 1 h on a PDP II/70. Thus for the full first-order solutions of equations (16), depicted in Figures 1–3, the computational time was $2\frac{1}{2}$ h. When the optically thick gas approximation is made, the time reduces to $1\frac{1}{6}$ h, with a maximum error of 5% on the values in Figures 1–3.

Finally, we discuss the errors involved in the finite difference scheme and the convergence of the asymptotic series expansion. First of all we compare the solutions of equation (10a) for δ small with the corresponding finite difference results on equations leading to (10a). In principle we keep D_m fixed and choose M such that $\delta \ll 1$. We find that when $\delta = 0{\cdot}1$ the asymptotic and finite difference solutions (for $y = 0{\cdot}1$) differ by 2·7%. If δ is further reduced to 0·01, this difference becomes 1·5%. In either case the finite difference solution is larger than the asymptotic one. Also if the numerical experiment is re-run with $\varepsilon = 0{\cdot}01$ rather than 0·1, but with the other parameters kept fixed as given above, u, θ and h increase by less than 0·5%. We conclude that both the finite difference scheme and the two-term asymptotic series representation give stable and convergent solutions.

APPENDIX

We give a brief account of the polynomial solutions for temperature in the order $O(1)$ case given in Section 3. For the general differential approximation for a grey gas the one-dimensional equation for the flux is

$$\frac{d^2 q}{dy^2} - \frac{3}{N^2} q - \frac{3}{B_0} \theta^3 \frac{d\theta}{dy} = 0, \tag{18}$$

in which the temperature is given by

$$0=\frac{1}{Pr}\frac{\mathrm{d}}{\mathrm{d}y}\left(\theta\frac{\mathrm{d}\theta}{\mathrm{d}y}\right)-\frac{\mathrm{d}q}{\mathrm{d}y}. \tag{19}$$

We consider the following cases.

(i) *Optically thin* ($\alpha_r \ll 1$ *and* N *large*)

The approximation for the flux is

$$\frac{\mathrm{d}q}{\mathrm{d}y}=\frac{3}{4B_0}(\theta^4-1),$$

which together with the temperature equation results in a polynomial of sixth degree in the determination of the temperature.

(ii) *Optically thick* ($\alpha_r \gg 1$ *and* N *small*)

It is usual to discard the $\mathrm{d}^2q/\mathrm{d}y^2$ term in the general flux equation for this approximation. However, when the space is semi-infinite, this approximation may prove troublesome except for flows with suction.[8] In this case we replace

$$\frac{\mathrm{d}^2q}{\mathrm{d}y^2}-\frac{3}{N^2}q$$

by

$$-\frac{3}{N^2}\frac{\mathrm{d}q}{\mathrm{d}y}$$

to get

$$\frac{\mathrm{d}q}{\mathrm{d}y}=-\frac{N^2}{B_0}\theta^3\frac{\mathrm{d}\theta}{\mathrm{d}y},$$

and this equation with (19) gives rise to a quartic equation.

(iii) *The general case*

This is tackled in the spirit of Bestman.[7,8]

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